DEGREE OF OUTLIER CALCULATION DEVICE, AND PROBABILITY DENSITY ESTIMATION DEVICE AND HISTOGRAM CALCULATION DEVICE FOR USE THEREIN

BACKGROUND OF THE INVENTION

FIELD OF THE INVENTION

The present invention relates to a degree of outlier calculation device, and a probability density estimation device and a histogram calculation device for use therein and, more particularly, to statistical outlier detection, fraud detection and fraud detection techniques for detecting an abnormal value or an outlier which largely deviates from data patterns obtained so far from multi-dimensional time series data.

DESCRIPTION OF THE RELATED ART

Such a degree of outlier calculation device is for use in finding an abnormal value or an outlier which largely deviates from data patterns obtained so far from multi-dimensional time series data and is employed, for example, in a case of finding such fraud behavior as so-called cloning use from a record of cellular phone services and in a case of finding abnormal transaction from a use history of a credit card.

Well-known conventional fraud detection methods using a machine learning technique include the method by T. Fawcett and F Provost ("Combining Data Mining and

Machine Learning for Effective Fraud Detection, Proceedings of AI Approaches to Fraud Detection and Risk Management, pp. 14-19, 1997") and the method by J. Ryan, M. Lin and R. Miikkulainen ("Intrusion Detection with Neural Networks, Proceedings of AI Approaches to Fraud Detection and Risk Management, pp. 72-77, 1997").

Among the above methods, one that makes use of an idea of statistical outlier detection, in particular, is the method by P. Burge and J. Shawe-Taylor ("Detecting Cellular Fraud Using Adaptive Prototypes, Proceedings of AI Approaches to Fraud Detection and Risk Management, pp. 9-13, 1997").

As a learning algorithm for a parametric finite mixture model, well-known is the EM Algorithm by A.P. Dempster, N.M Laird and D.B. Ribin ("Maximum Likelihood from Incomplete Data via the EM Algorithm, Journal of the Royal Statistical Society, B, 39(1), pp. 1-38, 1977").

As a learning algorithm for a normal kernel mixture distribution (a mixture of a finite number of the same normal distributions), the prototype updating algorithm by I. Grabec is known ("Self-Organization of Neurons Described by the Maximum-Entropy Principle, Biological Cybernetics, vol. 63, pp. 403-409, 1990").

The above-described methods by T. Fawcett and F. Provost and by J. Ryan, M. Lin and R. Miikkulainen relate to fraud detection realized by learning unfair

detection patterns from data whose fraud is known (so-called supervised data). In practice, however, it is so difficult to obtain sufficient unfair data that highly precise learning can not be conducted to result in a decrease in fraud detection precision.

The method by P. Burge and J. Shawe-Taylor relates to similar fraud detection based on unsupervised data. This method, however, conducts fraud detection with two non-parametric models, a short-term model and a long-term model, to make a distance between them as a criterion for an outlier. Statistical basis of the short-term model and the long-term model is insufficient to make statistical significance of a distance therebetween unclear.

In addition, preparation of two models, short-term and long-term models, deteriorates calculation efficiency. Further problems are involved such as a problem that only continuous value data can be handled and not categorical data and a problem that since only non-parametric models are handled, fraud detection is unstable and inefficient.

Although as a learning algorithm for a statistical model, the EM algorithm by A.P. Dempster, N.M. Laird and D.B. Ribin and the prototype updating algorithm by I. Grabec are known, since these algorithms learn from all the past data equally weighted , they fail to cope with a pattern change.

SUMMARY OF THE INVENTION

An object of the present invention is to provide a degree of outlier calculation device capable of automatically detecting fraud based on data whose fraud is yet to be known (unsupervised data), and a probability density estimation device and a histogram calculation device for use therein.

Another object of the present invention is to provide a degree of outlier calculation device which adopts an outlier determination criteria whose statistical significance is clear and uses a model including short-term and long-term models combined into one, thereby improving efficiency of calculation, coping with categorical data and enabling stable and efficient outlier detection using not only a non-parametric model but also a parametric model, and a probability density estimation device and a histogram calculation device for use therein.

A further object of the present invention is to provide a degree of outlier calculation device which realizes in the device an algorithm learning while forgetting past data by weighting less on older data to enable even a change in pattern to be flexibly followed, and a probability density estimation device and a histogram calculation device for use therein.

According to the first aspect of the invention,

for use in a degree of outlier calculation device for
sequentially calculating a degree of outlier of each
data with a data sequence of real vector values as input,
a probability density estimation device for, while
sequentially reading the data sequence, estimating a
probability distribution of the data in question by
using a finite mixture of normal distributions (normal
mixture for short), comprises

probability calculation means for calculating,
based on a value of input data and values of a mean
parameter and a variance parameter of each of a finite
number of normal distribution densities, a probability
of generation of the input data in question from each
normal distribution, and

parameter rewriting means for updating and
rewriting the stored parameter values while forgetting
past data, according to newly read data based on a
probability obtained by the probability calculation
means, values of a mean parameter and a variance
parameter of each normal distribution and a weighting
parameter of each normal distribution.

In the preferred construction, the probability
density estimation device further comprises

parameter storage means for storing values of a
mean parameter and a variance parameter of each of a
finite number of normal distribution densities and a
weighting parameter of each normal distribution, wherein

the parameter rewriting means updates and rewrites data of the parameter storage means.

According to the second aspect of the invention, a degree of outlier calculation device for sequentially detecting a degree of outlier of each data with a data sequence of real vector values as input, comprises

a probability density estimation device for, while sequentially reading the data sequence, estimating a probability distribution of generation of the data in question by using a finite mixture of normal distributions including

(a) parameter storage means for storing values of a mean parameter and a variance parameter of each of a finite number of normal distribution densities and a weighting parameter of each normal distribution,

(b) probability calculation means for calculating, based on a value of input data and values of a mean parameter and a variance parameter of each of a finite number of normal distribution densities, a probability of generation of the input data in question from each normal distribution, and

(c) parameter rewriting means for updating and rewriting the stored parameter values while forgetting past data, according to newly read data based on a probability obtained by the probability calculation means, values of a mean parameter and a variance parameter of each normal distribution and a weighting

parameter of each normal distribution, and

degree of outlier calculation means for calculating and outputting a degree of outlier of the data by using a parameter of the normal mixture updated by the probability density estimation device and based on a probability distribution estimated from values of the parameters before and after the updating and the input data.

According to the third aspect of the invention, a probability density estimation device for use in a degree of outlier calculation device to, while sequentially reading a data sequence, estimate a probability distribution of generation of the data in question by using a finite number of normal kernel distributions, comprises

parameter storage means for storing a value of a parameter indicative of a position of each kernel, and

parameter rewriting means for reading a value of a parameter from the storage means and updating the stored parameter values while forgetting past data, according to newly read data to rewrite the contents of the parameter storage means.

According to another aspect of the invention, a degree of outlier calculation device for sequentially calculating a degree of outlier of each data with a data sequence of real vector values as input, comprises

a probability density estimation device for,

while sequentially reading the data sequence, estimating
a probability distribution of generation of the data in
question by using a finite number of normal kernel
distributions including

(a) parameter storage means for storing a value
of a parameter indicative of a position of each kernel,
and

(b) parameter rewriting means for reading a value
of a parameter from the storage means and updating the
stored parameter values while forgetting past data,
according to newly read data to rewrite the contents of
the parameter storage means, and

degree of outlier calculation means for
calculating and outputting a degree of outlier of the
data by using the parameter updated by the probability
density estimation device and based on a probability
distribution estimated from values of the parameters
before and after the updating and the input data.

According to another aspect of the invention, for
use in a degree of outlier calculation device for
sequentially calculating a degree of outlier of each
data with discrete value data as input, a histogram
calculation device for calculating a parameter of a
histogram with respect to the discrete value data
sequentially input, comprises

storage means for storing a parameter value of
the histogram, and

parameter updating means for reading the parameter value from the storage means and updating past parameter values while forgetting past data based on input data to rewrite the value of the storage means, thereby outputting some of parameter values of the storage means.

According to another aspect of the invention, a degree of outlier calculation device for sequentially calculating a degree of outlier of each data with discrete value data as input, comprises

a histogram calculation device for calculating a parameter of a histogram with respect to the discrete value data sequentially input including

storage means for storing a parameter value of the histogram, and

parameter updating means for reading the parameter value from the storage means and updating past parameter values while forgetting past data based on input data to rewrite the value of the storage means, thereby outputting some of parameter values of the storage means, and

score calculation means for calculating, based on the output of the histogram calculation device and the input data, a score of the input data in question with respect to the histogram, thereby outputting the output of the score calculation means as a degree of outlier of the input data.

According to another aspect of the invention, a degree of outlier calculation device for calculating a degree of outlier with respect to sequentially input data which is described both in a discrete value and in a continuous value , comprises

a histogram calculation device for estimating a histogram with respect to a discrete value data part,

probability density estimation devices provided as many as the number of cells of the histogram for estimating a probability density with respect to a continuous value data part,

cell determination means for determining to which cell of the histogram the discrete value data part belongs to send the continuous data part to the corresponding one of the probability density estimation devices, and

score calculation means for calculating a score of the input data based on a probability distribution estimated from output values of the histogram calculation device and the probability density estimation device and the input data, thereby

outputting the output of the score calculation means as a degree of outlier of the input data,

the histogram calculation device including

storage means for storing a parameter value of the histogram, and

parameter updating means for reading the

parameter value from the storage means and updating past

parameter values while forgetting past data based on

input data to rewrite the value of the storage means,

thereby outputting some of parameter values of the

5   storage means, and

the probability density estimation device

including

parameter storage means for storing values of a

mean parameter and a variance parameter of each of a

10   finite number of normal distribution densities and a

weighting parameter of each normal distribution,

probability calculation means for calculating,

based on a value of input data, and values of a mean

parameter and a variance parameter of each of a finite

15   number of normal distribution densities, a probability

of generation of the input data in question from each

normal distribution, and

parameter rewriting means for updating and

rewriting the stored parameter values while forgetting

20   past data, according to newly read data based on a

probability obtained by the probability calculation

means, values of a mean parameter and a variance

parameter of each normal distribution and a weighting

parameter of each normal distribution.

25   According to another aspect of the invention, a

degree of outlier calculation device for calculating a

degree of outlier with respect to sequentially input

data which is described both in a discrete value and in a continuous value , comprises

a histogram calculation device for estimating a histogram with respect to the discrete value data part,

probability density estimation devices provided as many as the number of cells of the histogram for estimating a probability density with respect to a continuous value data part,

cell determination means for determining to which cell of the histogram the discrete value data part belongs to send the continuous data part to the corresponding one of the probability density estimation devices, and

score calculation means for calculating a score of the input data based on a probability distribution estimated from output values of the histogram calculation device and the probability density estimation device and the input data, thereby

outputting the output of the score calculation means as a degree of outlier of the input data,

the histogram calculation device including

storage means for storing a parameter value of the histogram, and

parameter updating means for reading the parameter value from the storage means and updating past parameter values while forgetting past data based on input data to rewrite the value of the storage means,

thereby outputting some of parameter values of the storage means, and

the probability density estimation device including

parameter storage means for storing a value of a parameter indicative of a position of each kernel, and

parameter rewriting means for reading a value of a parameter from the storage means and updating the stored parameter values while forgetting past data, according to newly read data to rewrite the contents of the parameter storage means.

According to another aspect of the invention, for use in a degree of outlier calculation device for sequentially calculating a degree of outlier of each data with a data sequence of real vector values as input, a probability density estimation method of, while sequentially reading the data sequence, estimating a probability distribution of generation of the data in question by using a finite mixture of normal distributions, comprising the steps of

based on values of a mean parameter and a variance parameter of each of a finite number of normal distribution densities read from parameter storage means for storing a value of input data, values of a mean parameter and a variance parameter of each of a finite number of normal distribution densities, and a weighting parameter of each normal distribution, calculating a

probability of generation of the input data in question from each normal distribution, and

updating the stored parameter values while forgetting past data, according to newly read data based on a probability obtained by the probability calculation means, values of a mean parameter and a variance parameter of each normal distribution and a weighting parameter of each normal distribution to rewrite data of the parameter storage means.

According to another aspect of the invention, a degree of outlier calculation method of sequentially calculating a degree of outlier of each data, with a data sequence of real vector values as input, wherein

probability density estimation for, while sequentially reading the data sequence, estimating a probability distribution of generation of the data in question by using a finite mixture of normal distributions, comprises the steps of:

based on values of a mean parameter and a variance parameter of each of a finite number of normal distribution densities read from parameter storage means for storing a value of input data, values of a mean parameter and a variance parameter of each of a finite number of normal distribution densities, and a weighting parameter of each normal distribution, calculating a probability of generation of the input data in question from each normal distribution, and

updating the stored parameter values while forgetting past data, according to newly read data based on a probability obtained by the probability calculation means, values of a mean parameter and a variance parameter of each normal distribution and a weighting parameter of each normal distribution to rewrite data of the parameter storage means, and which further comprises the step of:

calculating and outputting a degree of outlier of the data by using a parameter of the normal mixture updated by the probability density estimation and based on a probability distribution estimated from values of the parameters before and after the updating and the input data.

According to another aspect of the invention, a probability density estimation method for use in calculation of a degree of outlier to, while sequentially reading a data sequence, estimate a probability distribution of generation of the data in question by using a finite number of normal kernel distributions, comprising the steps of:

storing a value of a parameter indicative of a position of each kernel in parameter storage means, and

reading a value of a parameter from the storage means and updating the stored parameter values while forgetting past data, according to newly read data to rewrite the contents of the parameter storage means.

According to another aspect of the invention, a degree of outlier calculation method of sequentially calculating a degree of outlier of each data, with a data sequence of real vector values as input, wherein

probability density estimation for, while sequentially reading the data sequence, estimating a probability distribution of generation of the data in question by using a finite number of normal kernel distributions comprises the steps of:

storing a value of a parameter indicative of a position of each kernel in parameter storage means,

reading a value of a parameter from the storage means and updating the stored parameter values while forgetting past data, according to newly read data to rewrite the contents of the parameter storage means, and which further comprises:

degree of outlier calculation means for calculating and outputting a degree of outlier of the data by using the parameter updated by the probability density estimation and based on a probability distribution estimated from values of the parameters before and after the updating and the input data.

According to another aspect of the invention, for use in calculation of a degree of outlier for sequentially calculating a degree of outlier of each data with discrete value data as input, a histogram calculation method of calculating a parameter of a

histogram with respect to the discrete value data
sequentially input, comprising the steps of:

reading the parameter value from storage means
for storing a parameter value of the histogram and
5 updating past parameter values while forgetting past
data based on input data to rewrite the value of the
storage means, and

outputting some of parameter values of the
storage means.

10 According to a further aspect of the invention, a
degree of outlier calculation device for sequentially
calculating a degree of outlier of each data with
discrete value data as input, comprising:

a histogram calculation device for calculating a
15 parameter of a histogram with respect to the discrete
value data sequentially input including

storage means for storing a parameter value of
the histogram, and

parameter updating means for reading the
20 parameter value from the storage means and updating past
parameter values while forgetting past data based on
input data to rewrite the value of the storage means,
thereby outputting some of parameter values of the
storage means, and

25 score calculation means for calculating, based on
the output of the histogram calculation device and the
input data, a score of the input data in question with

respect to the histogram, thereby outputting the score calculation result as a degree of outlier of the input data.

According to a still further aspect of the invention, a degree of outlier calculation method of calculating a degree of outlier with respect to sequentially input data which is described both in a discrete value and in a continuous value , wherein

histogram calculation which estimates a histogram with respect to a discrete value data part comprises the steps of:

reading the parameter value from storage means for storing a parameter value of the histogram and updating past parameter values while forgetting past data based on input data to rewrite the value of the storage means, and

outputting some of parameter values of the storage means, and wherein

in probability density estimation devices provided as many as the number of cells of the histogram for estimating a probability density with respect to a continuous value data part, the method comprises the steps of:

based on values of a mean parameter and a variance parameter of each of a finite number of normal distribution densities read from parameter storage means for storing a value of input data, values of a mean

parameter and variance parameter of each of a finite

number of normal distribution densities and a weighting

parameter of each normal distribution, calculating a

probability of generation of the input data in question

5   from each normal distribution, and

based on a probability obtained by the

probability calculation means, values of a mean

parameter and a variance parameter of each normal

distribution and a weighting parameter of each normal

10   distribution, updating the stored parameter values while

forgetting past data, according to newly read data to

rewrite the data of the parameter storage means, and

wherein the method further comprises the steps of:

determining to which cell of the histogram the

15   discrete value data part belongs to send the continuous

data part to the corresponding one of the probability

density estimation devices,

calculating a score of the input data based on a

probability distribution estimated from output values of

20   the histogram calculation device and the probability

density estimation device and the input data, and

outputting the score calculation result as a

degree of outlier of the input data.

In the present invention, with one value of time

25   series data as x, assuming that input data is multi-

divisional data, the contents of x include, for example,

one real number, an attribute of a discrete value of a

multi-divisional real number value vector and a multi-divisional vector having the foregoing elements. In a case of cellular phone, x may be expressed as follows which is one example only:

5   x = (telephone service start time, telephone service duration time and origin of service)

A probability density function of a probability distribution followed by x represents character of a data generation mechanism (e.g. telephone service

10   pattern of user). The degree of outlier calculation device according to the present invention learns a probability density function every time time series data is applied. Under these circumstances, it is assumed that a "degree of outlier" is basically calculated based

15   on the two ideas (A) and (B) shown below.

A) A degree of outlier of one input data is calculated based on the amount of a change in a learned probability density from that before learning caused as a result of taking in the input data. This is on the premise that

20   data largely differing in tendency from a learned probability density function is considered to have a high degree of outlier. More specifically, a function of a distance between probability densities before and after data input is calculated as a degree of outlier.

25   B) A likelihood of a probability density function so far obtained by learning with respect to input data is calculated (value of the probability density function

with respect to the input data). It can be understood
that larger the likelihood is, the higher the degree of
outlier is. In practice, a value obtained by adding a
negative sign to a logarithm of the likelihood (negative
logarithmic likelihood) is output as a degree of outlier.

In addition, a combination of the above two
functions and the like can be used. As described in the
foregoing, the device according to the present invention
represents statistical character of a data generation
mechanism by a probability density function (the
function of a probability density estimation device) and
based thereon, calculates and outputs how input data
deviates from the character of the data generation
mechanism as a "degree of outlier" (the function of the
degree of outlier calculation device).

Other objects, features and advantages of the
present invention will become clear from the detailed
description given herebelow.

BRIEF DESCRIPTION OF THE DRAWINGS

The present invention will be understood more
fully from the detailed description given herebelow and
from the accompanying drawings of the preferred
embodiment of the invention, which, however, should not
be taken to be limitative to the invention, but are for
explanation and understanding only.

In the drawings:

Fig. 1 is a diagram showing a structure of one example of a probability density estimation device (normal mixture) according to the present invention;

Fig. 2 is a flow chart showing operation of the device illustrated in Fig. 1;

Fig. 3 is a diagram showing a structure of an example of a degree of outlier calculation device using the device of Fig. 1;

Fig. 4 is a flow chart of operation of the device illustrated in Fig. 3;

Fig. 5 is a diagram showing a structure of one example of a probability density estimation device (kernel mixture) according to the present invention;

Fig. 6 is a flow chart of operation of the device illustrated in Fig. 5;

Fig. 7 is a diagram showing a structure of an example of a degree of outlier calculation device using the device of Fig. 6;

Fig. 8 is a flow chart of operation of the device illustrated in Fig. 7;

Fig. 9 is a diagram showing a structure of one example of a histogram calculation device according to the present invention;

Fig. 10 is a flow chart of operation of the device illustrated in Fig. 9;

Fig. 11 is a diagram showing a structure of an example of a degree of outlier calculation device using

the device of Fig. 10;

Fig. 12 is a flow chart of operation of the device illustrated in Fig. 11;

Fig. 13 is a diagram showing a structure of an example of a degree of outlier calculation device using the devices of Figs. 1 and 9;

Fig. 14 is a flow chart of operation of the device illustrated in Fig. 13;

Fig. 15 is a diagram showing a structure of an example of a degree of outlier calculation device using the devices of Figs. 5 and 9;

Fig. 16 is a flow chart of operation of the device illustrated in Fig. 15.

DESCRIPTION OF THE PREFERRED EMBODIMENT

The preferred embodiment of the present invention will be discussed hereinafter in detail with reference to the accompanying drawings. In the following description, numerous specific details are set forth in order to provide a thorough understanding of the present invention. It will be obvious, however, to those skilled in the art that the present invention may be practiced without these specific details. In other instance, well-known structures are not shown in detail in order to unnecessary obscure the present invention.

First, description will be made of a probability density estimation device using a normal mixture. Assume

that data x (d-dimensional vector value) is generated
according to the following Expression 1 as a probability
distribution:

5

$$p(x|\theta) = \sum_{i=1}^{k} c_i p(x|\mu_i, \Sigma_i) \quad ------(1)$$

In the expression, holds the following:

$$p(x|\mu_i, \Sigma_i) = \frac{1}{(2\pi)^{d/2}|\Sigma_i|^{1/2}} \exp(-\frac{1}{2}(x - \mu_i)^T \sum_i^{-1}(x - \mu_i))$$

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and μ_i denotes a n-dimensional vector which is a
parameter indicative of a mean value of an n-dimensional
normal distribution and Σ_i denotes an n-dimensional
square matrix which is a parameter indicative of a
15 variance of the n-dimensional normal distribution. c_i
denotes a parameter indicative of a weight of a normal
distribution. Here, k represents an integer indicative
of the number of overlaps and holds the following:

20      $$c_i = 0 \quad \text{and} \quad \sum_{-1}^{k} c_i \geq 1$$

It is also assumed that $\theta = (c_i, \mu_i, \Sigma_i, ..., c_k, \mu_k, \Sigma_k)$
represents a parameter vector.

Fig. 1 is a block diagram showing a probability

density estimation device according to one embodiment of the present embodiment. Assume here that a constant r (0 $\leq$ r $\leq$ 1 and the smaller r becomes, the faster past data is forgotten) indicative of a forgetting speed and k as the number of overlaps of normal distributions are given in advance. In addition, the parameter α ($\alpha > 0$) is also used which is assumed to be given in advance.

In Fig. 1, a parameter storage device 13 is a device for storing the above-described parameter θ , a parameter rewriting device 12 is capable of storing a d-dimensional vector μ_i' and a d-dimensional square matrix Σ_i' as well. The reference numeral 10 represents a data input unit, 11 a probability calculation device for calculating a probability and 14 a parameter output unit.

Fig. 2 is a flow chart showing schematic operation of the block illustrated in Fig. 1 and the device of Fig. 1 operates in a manner as described in the following. First, initialize a value of each parameter stored in the parameter storage device 13 before data reading (Step S10). Next, the device operates in the following manner every time t-th data x_t is input. The input x_t is transferred to and stored in the probability calculation device 11 and the parameter rewriting device 12 (Step 11).

The probability calculation device 11 reads a current value θ of the parameter from the parameter storage device 13, based on the value, calculates each

probability γ_i ($i = 1, 2, \ldots, k$) that each normal distribution generates the data x_t according to the following [Expression 4] (Step S12) and sends the calculation result to the parameter rewriting device 12:

5

$$\gamma_i := (1 - \alpha r) \frac{c_i\, p(x_t | \mu_i, \Sigma_i)}{\sum_{i=1}^{k} c_i\, p(x_t | \mu_i, \Sigma_i)} + \frac{\alpha r}{k}$$

The parameter rewriting device 12 reads the current parameter value from the parameter storage device 13 while sequentially calculating an updating result of the

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parameter value with respect to each of $i = 1, 2, \ldots, k$ in a manner as shown in the following expressions (2) to (6) by using the received probability γ_i to rewrite the parameter values stored in the parameter storage device

15

13 (Step S13). In these expressions (2) to (6), the sign ":=" signifies that a right-side term is to substitute for a left-side term.

$$c_i := (1 - r)c_i + r\gamma_i \quad -----(2)$$
$$\mu_i := (1 - r)\mu_i' + r\gamma_i \cdot x_t \quad -----(3)$$
$$\mu_i := \frac{\mu_i'}{c_i} \quad -----(4)$$
$$\Sigma_i' := (1 - r)\Sigma_i' + r\gamma_i \cdot x_t x_t^T \quad -----(5)$$
$$\Sigma_i := \frac{\Sigma_i'}{c_i} - \mu_i \mu_i^T \quad -----(6)$$

20

Then, the parameter storage device 13 outputs the

rewritten parameter values (Step S14). The updating rule
is equivalent to maximization of a logarithmic
likelihood having a weight of $(1-r)^l$ with respect to the
(t-l)th data and realizes such estimation as made by
forgetting past data one by one. This accordingly
results in learning using latest 1/r number of data (l:
positive integer).

This is because a solution of $(1-r)^l = 1/2$ is
expressed as:

$$l = - (\log 2)/\log(1-r) \sim (\log 2)/r$$

Thus, the probability density expressed by the
above Expression (1) and the function is completely
designated by a finite number of parameters. Therefore,
only the designation of a parameter value is enough for
expressing the present probability density function, so
that the parameter output unit 14 illustrated in Fig. 1
enables estimation of the probability density function
in question. A device for calculating a degree of
outlier of input data using thus estimated probability
density function is shown in the block diagram of Fig. 3.

Fig. 3 is a block diagram showing one embodiment
of a degree of outlier calculation device. The present
device includes an input unit 20, a probability density
estimation device 21 illustrated in Fig. 1, a score
calculation device 22 for calculating a degree of
outlier of data, that is, a score, based on a

probability distribution estimated from input data and a parameter from the probability density estimation device 21, and an output unit 23 for outputting the calculation result. The device shown in Fig. 3 operates in the following manner according to a flow chart of Fig. 4 every time t-th data x_t is input.

The input x_t is transferred to the probability density estimation device 21 (normal mixture) and the score calculation device 22 (Step S20) and stored therein. The probability density estimation device 21 updates a value of a stored parameter according to the input data (Step S21) and inputs the new value to the score calculation device 22. The score calculation device 22 calculates a score using the input data, the parameter value and the parameter value handed over in the past (Step S22) and outputs the same (Step S23). A score indicative of a degree of outlier is calculated, for example, using a square distance, a Hellinger distance and further a logarithmic loss.

In the following, the calculation will be described more specifically. In a case where with a parameter $\theta(t)$ estimated from data $x_t = x_1 \, x_2 \, \ldots \, x_t$, the expression $p^{(t)}(x) = p(x|\theta(t))$ holds and with respect to probability distributions p and q, $d_s(p, q)$ represents a square distance between the two distributions and $dh(p,q)$ represents a Hellinger distance, any of the followings can be used as a score:

$$d_s(p^{(t)}, p^{(t-1)}) = \int \left(p^{(t)}(x) - p^{(t-1)}(x)\right)^2 dx$$

$$d_h(p^{(t)}, p^{(t-1)}) = \int \left(\sqrt{p^{(t)}(x)} - \sqrt{p^{(t-1)}(x)}\right)^2 dx$$

A logarithmic loss can be calculated by the following expression:

- logp (t - 1)(xt)

These can be immediately generalized into ds ($p^{(t)}$, $p^{(t-T)}$) etc. with T as a positive integer.

Next, another embodiment of a probability density estimation device according to the present invention will be described. In this example, used as a data generation model is the following expression which is a kernel mixture distribution:

$$p(x|q) = \frac{1}{k} \sum_{i=1}^{k} \omega(x : q_i)$$

In the expression, "$\omega(\cdot : \cdot)$ is called a kernel function which is provided in the form of the following normal density function (referred to as normal distribution kernel):

$$\omega(x:\omega_t) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-q_i)^T\Sigma^{-1}(x-q_i)\right)$$

In the expression, Σ represents a diagonal matrix and the following equation holds:

$$\Sigma = \text{diag} (\ \sigma^2,\ ...,\ \sigma^2)$$

σ represents an applied positive integer. Each q_i denotes a d-dimensional vector which is a parameter designating a position of each kernel function. $\{q_i\}$ is called prototype. x_m represents an m-th component of x. Similarly, qim represents an m-th component of q_i.

Fig. 5 is a block diagram showing a probability density estimation device using a kernel mixture distribution. A parameter storage device 32 has a function of storing $q = (q_1,\ q_2,\ ...\ q_k)$. In Fig. 5, 30 denotes an input unit, 31 a parameter rewriting device and 33 an output unit. The device shown in Fig. 5 operates in the following manner according to a flow chart of Fig. 6. First, prior to data reading, initialize a parameter value stored in the parameter storage device 32 (Step S30). Then, every time t-th data x_t is input, the device operates according to the following procedures. The input x_t is transferred to the parameter rewriting device 31 (Step S31) and stored therein. The parameter rewriting device 31 reads a current parameter value q from the parameter storage device 32 and obtains a solution Δq of the following

simultaneous linear equations (k = 1, 2, ..., K, 1 = 1, 2, ..., d) (δml represents a Kronecker delta, that is, when m = 1, it equals 1 and otherwise equals 0) to rewrite, as q := q+$\triangle$q, the parameter value stored in the parameter storage device 32 (Step S32):

$$\sum_{j=1}^{K} \sum_{m=1}^{d} C_{jmkl} \Delta_{qjm} = rB_{kl} \quad -----(7)$$

however

$$B_{kl} = K \cdot (x_t + 1, 1 - q_{kl}) \exp(-\frac{\left| x_{t+1} - q_k \right|^2}{4\sigma^2}) - \sum_{i=1}^{K} (q_{il} - q_{kl}) \exp(-\frac{\left| q_i - q_k \right|^2}{4\sigma^2})$$

$$C_{jmkl} = (\delta_{ml} - \frac{(q_{kl} - q_{jl})(q_{km} - q_{jm})}{2\sigma^2}) \exp(-\frac{\left| q_k - q_j \right|^2}{4\sigma^2})$$

The parameter storage device 32 outputs the rewritten parameter value (Step S33).

In the foregoing updating rules, r denotes a parameter which controls a forgetting speed. More specifically, a kernel mixture distribution obtained by sequentially adapting the rules in question minimizes a square distance from a probability density expressed as the following expression:

$$\sum_{\tau=2}^{t} r(1-r)^{t-\tau} w(x:x_\tau) + (1-r)^{t-1} w(x:x_1) \quad -----(8)$$

The algorithm by I. Grabec adopted by P. Burge and J. Shawe-Taylor corresponds to the above expression with r as a constant replaced by $1/\tau$. In this case, an expression corresponding to Expression (8) will be simply expressed as:

$$\sum_{\tau=1}^{t} (1/t) w(x:x_{\tau})$$

An example of a degree of outlier calculation device for calculating a degree of outlier of input data using a parameter obtained from the probability density estimation device employing a kernel mixture distribution shown in Fig. 5 is illustrated in Fig. 7. In Fig. 7, 40 represents an input unit, 41 the probability density estimation device shown in Fig. 5, 42 a score calculation device and 43 an output unit.

The device illustrated in Fig. 7 operates according to the following procedures and a flow chart of Fig. 8 every time t-th data x_t is input. The input x_t is transferred to the probability density estimation device 41 (kernel mixture distribution) and the score calculation device 42 (Step S40) and stored therein. The probability density estimation device 41 updates a value of a stored parameter according to the input data and

supplies the new value to the score calculation device
42. The score calculation device 42 calculates a score
using the input data, the value of the parameter and
values of parameters handed over in the past and outputs
5   the same (Steps S42 and S43). In this case, the same
score function as that in the degree of outlier
calculation device shown in Fig. 3 can be used.

Fig. 9 is a diagram showing an entire structure
of a histogram calculation device according to the
10   present invention. Discrete value data is sequentially
input to a parameter updating device 51 to which a
histogram storage device 52 is connected which stores a
parameter value of a histogram and outputs the same. 50
represents an input unit and 53 represents an output
15   unit.

Fig. 10 is a flow chart showing operation of the
device illustrated in Fig. 9. Assume that discrete value
data is designated by a number n of variables. Assume
here that an n-dimensional data space is divided into a
20   number N of exclusive cells in advance and that a
histogram is formed on these cells. Histogram represents
a probability distribution with $(p_1, \dots p_N)$ as a
parameter.

Here, p_j satisfies the following equation.

25

$$\sum_{j=1}^{N} p_j = 1, p_j \geq 0$$

Here, p_j represents an occurrence probability of a j-th cell. Assume that T0 (j) = 0 (j = 1, ..., N), 0<r<1 and β > 0 are given numbers and that initial parameters are as follows (Step S50):

$$p(0) (1) = \cdot \cdot \cdot \cdot \cdot \cdot = p(0) (N) = 1/N$$

The parameter updating device 51 conducts updating with respect to t-th input data [Step S51] in the following manner (Step S52):

$$T_t(j) = (1 - r)T_{t-1}(j) + \delta_t(j)$$
$$p^{(t)}(j) = \frac{T_t(j) + \beta}{(1 - (1 - r)^m)/r + N\beta}$$

In the expression , $\delta^t(j)$ takes 1 when the t-th data is input to the j-th cell and otherwise takes 0. This updating is conducted with respect to all the cells.

With $p^{(t)}$ (1), ..., $p^{(t)}$ (N) as new parameters of the histogram, updating is conducted. These values are sent to the histogram storage device 52. The histogram storage device 52 stores several past parameter values and outputs a part of them (Step S53).

The parameter updating device 51 conducts calculation at each step by multiplying data as of time t before by a weight of $(1-r)^t$. The weighting indicates that the older the data is, the more gradually it is

forgotten and realizes in the device an algorithm

learning while forgetting. As a result, it is possible

to flexibly follow a change of a user pattern.

A histogram represents a probability distribution

5   on a categorical variable and expresses, similarly to a

probability density function on a continuous variable,

statistical character of a data generation mechanism.

Accordingly, a relationship between the "histogram

calculation device" and the "degree of outlier

10   calculation device" is completely the same as that

between the above-described "probability density

estimation device" and "degree of outlier calculation

device". More specifically, the "histogram calculation

device" expresses statistical calculation of the data

15   generation mechanism based on which the "degree of

outlier calculation device" calculates how much input

data deviates from character of the data generation

mechanism as a "degree of outlier".

Fig. 11 shows an entire structure of a degree of

20   outlier calculation device using the histogram

calculation device illustrated in Fig. 9, and Fig. 12

shows a flow chart of the operation of the device.

Discrete value data from an input unit 60 is

sequentially input to a histogram calculation device 61

25   and a score calculation device 62 (Step S61). The score

calculation device 62 is connected to the histogram

calculation device 61 which outputs a parameter value of

the histogram from the input data (Step S62) and sends the same to the score calculation device 62. With the input data and the output of the histogram calculation device 61 as inputs, the score calculation device 62 calculates a score of a degree of outlier of the input data (Step S63).

As a score calculation method in this case, as well as in a case of continuous value data, a square distance, a Hellinger distance, a logarithmic loss, etc. can be used. In the histogram, a probability value $p^{(t)}(x)$ of data x to be stored in a j-th cell at a time t is calculated as follows:

$$p^{(t)}(x) = p^{(t)}(j)/L_j$$

In the expression, L_j denotes a number of points to be stored in the j-th cell and $p^{(t)}(j)$ denotes a probability value of the j-th cell at the time t. Using the equation, the square distance $ds(p^{(t)}, p^{(t-1)})$ and the Hellinger distance $dh(p^{(t)}, p^{(t-1)})$ are calculated according to the following expressions, respectively:

$$d_s(p^{(t)}, p^{(t-1)}) \overset{\text{def}}{=} \sum_x \left(p^{(t)}(x) - p^{(t-1)}(x) \right)^2,$$

$$d_h(p^{(t)}, p^{(t-1)}) \overset{\text{def}}{=} \sum_x \left(\sqrt{p^{(t)}(x)} - \sqrt{p^{(t-1)}(x)} \right)^2$$

For the score calculation device 62 to conduct these calculations, the degree of outlier calculation

device should be set to receive parameter values of $p^{(t)}$ and $p^{(t-1)}$ from the histogram calculation device 61. In addition, a logarithmic loss for input data x_t at a time t is calculated by the following expression:

5

$$- \log p^{(t-1)}(x_t)$$

The foregoing scores mean a change of an estimated distribution measured as a statistical distance or a logarithmic loss for an estimated

10   distribution of input data and either case their statistical significance is unclear.

Fig. 13 is a diagram showing an entire structure of a degree of outlier calculation device according to a further embodiment of the present invention which

15   employs the normal mixture density estimation device illustrated in Fig. 1 and the histogram calculation device illustrated in Fig. 9, while Fig. 14 is a flow chart showing operation thereof. Input data described both in a discrete value and a continuous value is

20   sequentially input to a histogram calculation device 71, a cell determination device 73 and a score calculation device 74 (Step S71). Connected to the cell determination device 73 are a number N of probability density calculation devices 721 to 72N for a normal

25   mixture. Here, N denotes the number of cells in the histogram of the histogram calculation device 71. To all the probability density calculation devices 721 to 72N

and the histogram calculation device 71, the score calculation device 74 is connected.

The histogram calculation device 71 calculates a parameter of the histogram only from a discrete data part of the input data (Step S72) and sends the same to the score calculation device 74. The cell determination device 73 determines to which cell of the histogram the discrete data part of the input data belongs (Step S73) and to the corresponding probability density estimation device, sends a continuous data part.

The probability density calculation devices 721 to 72N calculate a parameter of the probability density only when receiving the input data sent in (Step S74) and sends the parameter to the score calculation device 74. The score calculation device 74 calculates a score of the original input data with the input data, the output from the histogram calculation device 71 and any one of the outputs from the probability density calculation devices 721 to 72N as inputs (Step S75) and takes the score as an output (Step S76).

The score calculation device 74 calculates a score, for example, as a degree of a change in a probability distribution measured by a Hellinger distance or as a negative logarithmic likelihood (logarithmic loss) of a probability distribution with respect to input data. Denote a vector made up of categorical variables as x and a vector made up of

continuous variables as y. A simultaneous distribution
of x and y will be expressed as follows:

$$p(x, y) = p(x) \, p(y|x)$$

In the expression, p(x) represents a probability
distribution of x which is expressed by a histogram
density. p(y|x) represents a conditional probability
distribution of y with x being applied. This is provided
for each divisional region. With respect to new input
data Dt = (x_t, y_t), a Hellinger distance is calculated in
the following manner.

$$d_h(p^{(t)}, p^{(t-1)}) = 2 - 2 \sum_x \sqrt{p^{(t)}(x)p^{(t-1)}(x)} \int \sqrt{p^{(t)}(y|x)p^{(t-1)}(y|x)} \, dy$$

These are immediately generalized into a distance
between $p^{(t)}$ and $p^{(t-T)}$, with T as a positive integer.

In addition, a logarithmic loss is calculated
according to the following expression:

$$-\log p^{(t-1)}(x_t) - \log p^{(t-1)}(y_t|x_t)$$

Fig. 15 is a diagram showing an entire structure
of a degree of outlier calculation device according to
the present invention which employs the kernel mixture
distribution probability density estimation device
illustrated in Fig. 5 and the histogram calculation
device illustrated in Fig. 9, while Fig. 16 is a flow
chart showing operation thereof. Input data described

both in a discrete value and a continuous value the
stored parameter values is sequentially input to a
histogram calculation device 81, a cell determination
device 83 and a score calculation device 84 (Step S81).
To the cell determination device 83, a number N of
probability density calculation devices 821 to 82N for a
kernel mixture distribution are connected. Here, N
denotes the number of cells in the histogram of the
histogram calculation device 81.

To all the probability density calculation
devices 821 to 82N and the histogram calculation device
81, the score calculation device 84 is connected. The
histogram calculation device 81 calculates a parameter
of the histogram only from a discrete data part of the
input data (Step S82) and sends the same to the score
calculation device 84. The cell determination device 83
determines to which cell of the histogram the discrete
data part of the input data belongs (Step S83) and to
the corresponding probability density estimation device,
sends a continuous data part. The probability density
calculation devices 821 to 82N calculate a parameter of
the probability density only when receiving the input
data sent in (Step S84) and sends the parameter to the
score calculation device 84 (Step S85).

The score calculation device 84 calculates a
score of the original input data with the input data,
the output from the histogram calculation device 81 and

any one of the outputs from the probability density calculation devices 821 to 82N as inputs and takes the score as an output (Step S86). The score calculation method is the same as that of the degree of outlier calculation device shown in Fig. 13.

Although the invention has been illustrated and described with respect to exemplary embodiment thereof, it should be understood by those skilled in the art that the foregoing and various other changes, omissions and additions may be made therein and thereto, without departing from the spirit and scope of the present invention. Therefore, the present invention should not be understood as limited to the specific embodiment set out above but to include all possible embodiments which can be embodies within a scope encompassed and equivalents thereof with respect to the feature set out in the appended claims.